# Intertemporal Substitution

## Introduction

"If you raise the interest rate, agents will save more." bears some resemblence to "If you raise the price of X, people will buy less X." The latter is from the discussion of the Two Goods - Two Prices Model, which leads to a demand curve for a good. The interest rate can be thought as the price of current consumption because consuming rather than saving leads to foregone interest income.

The Intertemporal Substitution Model is an extension of the Two Goods - Two Prices Model. The two goods are replaced by one good in two different time periods. For example, X1 is food consumed in the present and X2 is food consumed in the future. The simplest assumption about the agent's endowment is that he or she has all his or her income in the present. Given that food cannot be stored for long periods of time, the only way to finance food consumption in the future is saving money in the present.

## The Model

By convention, present consumption X1 is on the horizontal axis and future consumption X2 is on the vertical axis. The price of the consumption good is denoted P1 for the first period and P2 for the second period. The simplest assumption is that P1 equals P2 so that the inflation rate is zero.

The agent's current saving is W - P1 X1. If this amount is deposited in an account earning interest at the rate R, then future consumption P2 X2 can be equal to ( 1 + R ) ( W - P1 X1 ). Rearranging these terms yields the budget constraint

$$W = P1\ X1 + P2\ X2 / ( 1 + R ).$$

The slope of the budget constraint depends on the interest rate R.

## Exercises

1. Consider an agent with an all of his income (endowment) in the first period. For R = 0%, draw the budget constraint and the indifference curve tangent to that budget constraint.

2. Draw the budget constraint and tangent indifference curve for R = 20% and R = 40%.

3. A more realistic case is an agent with income in both periods. The budget constraint is

$$W1 + W2 / ( 1 + R ) = P1\ X1 + P2\ X2 / ( 1 + R ).$$

For such an agent, draw the budget constraint and tangent indifference curve for R = 0%, R = 20%, and R = 40%.

4. As the interest rate increases, why does the agent shift his consumption toward the future?

# Saving for Retirement

## The Model

The Intertemporal Substitution Model assumes that an agent is simply given an endowment. We now consider the more realistic case that the agent has to work to produce an endowment. The locus of possible production points available to the agent is known as the Production Possibility Frontier (PPF).

Consider two periods: a first period, the working years, where production is relatively easy and a second period, retirement, where production is more difficult.

## Questions

1. Graph the PPF as described.

2. Assume that the agent has no access to financial markets. Find the best possible indifference curve reachable from the PPF. Record the agent's utility.

3. Assume the agent has access to financial markets. Find the best budget constraint reachable from the PPF and the best indifference curve reachable from the budget constraint. Record the agent's utility.

4. Is the agent better off with access to financial markets? Why?

# Investing in Education

## The Model

Suppose the two periods are young (when schooling is a possibility) and old (when working is certain). The agent faces a decision about how long to stay in school. The production possibility frontier shows the resulting tradeoff between young period and old period income.

## Questions

1. Draw the PPF as described.

2. Assume the agent has no access to financial markets. Find the best indifference curve reachable from that PPF. Record the agent's utility and young period income.

3. Assume the agent has access to financial markets. Find the best budget constraint reachable from the agent's PPF, and find the best indifference curve reachable from that budget constraint. Record the agent's utility and young period income.

4. Does access to financial markets induce the agent to stay in school longer? Why?