Two Goods - Two Prices

Introduction

If seems pretty intuitive that "If you raise the price of X, people will buy less X." Another way to put this is that the demand curve for X is downward sloping. The Two Goods - Two Prices Model provides a basis for these observations in terms of utility maximizing behavior.

If reality, agents consume many goods that have many prices. Two Goods - Two Prices is the simplest possible context for utility maximizing behavior based on relative prices. Given that paper, chalkboards, and computer screens are two dimensional, Two Goods - Two Prices is also the obvious candidate for a graphical presentation.

The Model

An agent consumes X1 and X2, receiving utility

$$U = X1 \bullet X2.$$

His or her budget constraint is

$$W = P1 \bullet X1 + P2 \bullet X2,$$

where P1 and P2 are the prices of the two goods and W is nominal wealth. Assume that the agent does not leave any wealth unspent.

Exercises

1. Start with P1 = 1, P2 = 1, and W = 10. Draw the agent's budget constraint and the indifference curve tangent to that budget constraint. What are the optimal values for X1 and X2?

2. Increase or decrease P1. Redraw the diagram. What happens to the optimal X1?

3. Graph the various values for P1 against the resulting values for X1, thus constructing the agent's demand curve for X1.

4. Draw the income and substitution effects for a large decrease in P1. Label these effects on your diagram.

   If you watched closely, you might have noticed that the quantity of X2 consumed

did not change when you changed P1. This is a particular property of the simple utility function above. The "translog" utility function

$$U = 0.5 \ln(X1) + 0.5 \ln(X2) + 0.1 \ln(x1)^2 - 0.15 \ln(X1) \ln(X2)$$

is considerably more complicated, and it does not have this property.

5. Start the Translog Utility Project, which changes to the translog utility function. Follow the steps to derive the demand curve for X1. Graph your results.

6. Does the optimal consumption of X2 depend on P1?